CBCS SCHEME

USN 18MATDIP31

Third Semester B.E. Degree Examination, Jan./Feb. 2021
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Prove that $(1+\cos\theta+i\sin\theta)^n+(1+\cos\theta-i\sin\theta)^n=2^{n+1}\cos^n\left(\frac{\theta}{2}\right)\cos\left(\frac{n\theta}{2}\right)$. (08 Marks)

b. Express $1-i\sqrt{3}$ in the polar form and hence find its modulus and amplitude. (06 Marks)

c. Find the argument of $\dfrac{1+\sqrt{3}i}{1-\sqrt{3}i}$. (06 Marks)

OR

2 a. If $\vec{A}=4\hat{i}+3\hat{j}+\hat{k}$ and $\vec{B}=2\hat{i}-\hat{j}+2\hat{k}$ find a unit vector $\hat{N}$ perpendicular to both $\vec{A}$ and $\vec{B}$ such that $\vec{A}$, $\vec{B}$ and N from a right handed system. (08 Marks)

b. If $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=3\hat{i}-\hat{j}+2\hat{k}$ then show that $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ are orthogonal. (06 Marks)

c. Show that the position vectors of the vertices of a triangle $\vec{A}=3(\sqrt{3}\,\hat{i}-\hat{j})$, $\vec{B}=6\hat{i}$ and $C=3(\sqrt{3}i+j)$ form an isosceles triangle. (06 Marks)

Module-2

3 a. Obtain the Maclaurin series expansion of log secx upto to the terms containing x^6. (08 Marks)

b. If $u=\tan^{-1}\left(\dfrac{x^3+y^3}{x-y}\right)$, prove that $xu_x+yu_y=\sin 2u$. (06 Marks)

c. If $u=f(x-y, y-z, z-x)$, show that $u_x+u_y+u_z=0$. (06 Marks)

OR

4 a. Prove that $\log(1+x)=x-\dfrac{x^2}{2}+\dfrac{x^3}{3}-\dfrac{x^4}{4}\ldots$ by using Maclaurin's series notation. (08 Marks)

b. Using Euler's theorem, prove that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=3u\log u$. If $u=e$ (06 Marks)

c. If $u=x+y$, $v=y+z$, $w=z+x$, find $J\left(\dfrac{u,v,w}{x,y,z}\right)$. (06 Marks)

Module-3

5 a. A particle moves along the curve $\vec{r}=\cos 2t\,\hat{i}+\sin 2t\,\hat{j}+t\,\hat{k}$. find the velocity and acceleration at $t=\dfrac{\pi}{8}$ along $\sqrt{2}\,\hat{i}+\sqrt{2}\,\hat{j}+\hat{k}$. (08 Marks)

b. Find the unit normal to the surface, $xy+x+zx=3$ at (1, 1, 1). (06 Marks)

c. Find the constant 'a' such that the vector field $\vec{F}=2xy^2z^2\hat{i}+2x^2yz^2\hat{j}+ax^2y^2z\hat{k}$ is irrotational. (06 Marks)

OR

6 a. If $\vec{F} = (x+y+1)\hat{i} + \hat{j} - (x+y)\hat{k}$ show that $\vec{F}.\text{curl}\,\vec{F} = 0$. **(08 Marks)**

b. If $\phi(x, y, z) = xy^2 + yz^3$, find $\nabla\phi$ & $|\nabla\phi|$ at $(1, -2, -1)$ **(06 Marks)**

c. Show that vector field $\vec{F} = \left[\dfrac{x\hat{i} + y\hat{j}}{x^2 + y^2}\right]$ is solenoidal. **(06 Marks)**

Module-4

7 a. Obtain a reduction for $\int_0^{\frac{\pi}{2}} \sin^n x\,dx$ $(n > 0)$. **(08 Marks)**

b. Evaluate $\int_0^1 \dfrac{x^9}{\sqrt{1-x^2}}dx$. **(06 Marks)**

c. Evaluate $\iint_R xy\,dx\,dy$ where R is the first quadrant of the circle $x^2 + y^2 = a^2$, $x \geq 0$, $y \geq 0$. **(06 Marks)**

OR

8 a. Obtain a reduction formula for $\int_0^{\frac{\pi}{2}} \cos^n x\,dx$, $(n > 0)$. **(08 Marks)**

b. Evaluate $\int_0^{2a} x^2\sqrt{2ax - x^2}\,dx$. **(06 Marks)**

c. Evaluate $\int_{-1}^{1}\int_0^{z}\int_{x-z}^{x+z}(x+y+z)\,dy\,dx\,dz$ **(06 Marks)**

Module-5

9 a. Solve $\dfrac{dy}{dx} + y\cot x = \sin x$. **(08 Marks)**

b. Solve $\cos x \sin y\,dx + \cos y \sin x\,dy = 0$. **(06 Marks)**

c. Solve $\dfrac{dy}{dx} + \dfrac{y}{x} = y^2 x$. **(06 Marks)**

OR

10 a. Solve : $\dfrac{dy}{dx} + \dfrac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$. **(08 Marks)**

b. Solve : $\dfrac{dy}{dx} + \dfrac{y}{x} = y^2 x$. **(06 Marks)**

c. Solve : $\sqrt{1-y^2}\,dx = (\sin^{-1} y - x)\,dy$ **(06 Marks)**

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